

# Student Workbook

## Law of Cosines

**Learning Target:** I can use the Law of Cosines to solve SAS and SSS triangles, and find their area with Heron's Formula.

### Vocabulary

| Term            | Meaning                                                                        |
|-----------------|--------------------------------------------------------------------------------|
| Law of Cosines  | $c^2 = a^2 + b^2 - 2ab \cos C$ — relates all three sides and one angle.        |
| SAS             | Two sides and the INCLUDED angle are known.                                    |
| SSS             | All three sides are known.                                                     |
| Heron's Formula | Area = $\sqrt{s(s-a)(s-b)(s-c)}$ , where $s = (a+b+c)/2$ (the semi-perimeter). |

### Key Concept: Solving SAS Triangles

**METHOD** Use the Law of Cosines to find the side OPPOSITE the given angle first, then use the Law of Sines to find a second angle.

#### EXAMPLE 1

$a=7$ ,  $b=9$ ,  $C=50^\circ$ . Find  $c$ .

$$c^2 = 49 + 81 - 2(7)(9)\cos(50^\circ) = 130 - 126\cos(50^\circ) \approx \underline{\hspace{2cm}} \rightarrow c \approx \underline{\hspace{2cm}}$$

### Key Concept: Solving SSS Triangles

**METHOD** Solve the Law of Cosines for cosine:  $\cos C = (a^2 + b^2 - c^2)/(2ab)$ . Start with the LARGEST side to find the largest angle first (helps catch obtuse triangles).

**EXAMPLE 2**

$a=8$ ,  $b=10$ ,  $c=13$ . Find angle  $C$  (opposite the largest side).

$$\cos C = (64+100-169)/(2 \cdot 8 \cdot 10) = \underline{\hspace{2cm}} \rightarrow C \approx \underline{\hspace{2cm}}$$

**COMMON MISTAKE**

Don't forget the "2" in " $-2ab \cdot \cos C$ "! Writing  $c^2 = a^2 + b^2 - ab \cdot \cos C$  (missing the 2) is one of the most common errors in this chapter.

**Key Concept: Heron's Formula****EXAMPLE 3**

Find the area of a triangle with  $a=8$ ,  $b=10$ ,  $c=13$  using Heron's Formula.

$$s = (8+10+13)/2 = \underline{\hspace{2cm}} \rightarrow \text{Area} = \sqrt{[15.5(7.5)(5.5)(2.5)]} \approx \underline{\hspace{2cm}}$$

**Guided Practice — Part A: Solve SAS (Find the Missing Side)**

**A1**  $a=6$ ,  $b=8$ ,  $C=60^\circ$ . Find  $c$ .

**A2**  $a=10$ ,  $b=14$ ,  $C=45^\circ$ . Find  $c$ .

**Guided Practice — Part B: Solve SSS (Find an Angle)**

**B1**  $a=5$ ,  $b=7$ ,  $c=9$ . Find angle  $C$ .

**B2**  $a=9$ ,  $b=12$ ,  $c=15$ . Find angle  $C$ .

### Guided Practice — Part C: Heron's Formula

**C1**  $a=7$ ,  $b=9$ ,  $c=12$ . Find the area.

**C2**  $a=6$ ,  $b=8$ ,  $c=10$ . Find the area.

### Guided Practice — Part D: Solve the Triangle Completely (SAS)

**D1**  $a=10$ ,  $b=15$ ,  $C=40^\circ$ . Find  $c$ , then angles  $A$  and  $B$ .

**D2**  $a=8$ ,  $b=6$ ,  $C=70^\circ$ . Find  $c$ , then angles  $A$  and  $B$ .

#### BEFORE YOU MOVE ON

Check with a partner: for SSS problems, did you find the **LARGEST** angle first (opposite the longest side)?